MA/MSCMT - 09

December - Examination 2015

M.A./M.Sc. Final Mathematics Examination

Integral Transforms and Integral Equations

Paper - MA/MSCMT - 09

Time : 3 Hours] **[Max. Marks :- 80**

Note : The question paper is divided into three Sections A, B, and C. Use of calculator is allowed in this paper.

Section - A

8 x 2 = 16

Note : Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1) (i) Define the convolution of two functions for Fourier transform.

(ii) Define Inverse Mellin transform.

(iii) Write change of scale property for Hankel transform.

(iv) Define the singular Integral equation.

(v) State second shifting property for Laplace transform.

(vi) Define Eigen values of a kernel in the integral equation.

(vii) Define the term "separable kernel".

(viii)Define complex Hilbert space.

Section - B 4 x 8 = 32

Note : Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Using Fredholm theory, solve

$$g(x) = \cos 2x + \int_0^{2\pi} \sin x \cos t \, g(t) dt$$

3) Find the resolvent kernel of Volterra integral equation with the kernel.

$$K(x,t) = \frac{(2 + \cos x)}{(2 + \cos t)}$$

4) Show that the integral equation

$$g(x) = f(x) + \frac{1}{\pi} \int_0^{2\pi} \sin(x + t) \, g(t) dt$$

possesses no solution for $f(x) = x$, but possesses infinitely may solutions when $f(x) = 1$.

5) Find the Hankel transform of order zero of $x^2 H(a - x)$.

6) State and prove convolution theorem for Mellin transform

7) Find Fourier Transform of

$$f(x) = \begin{cases} a - |x|, & for|x| \le a \\ 0, & for|x| > a \end{cases}$$

Hence prove that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$

8) Use convolution theorem to find inverse Laplace Transform of

$\frac{1}{p}\log\left(1 + \frac{1}{p^2}\right)$.

9) Find Laplace transform of $erfc\{\sqrt{t}\}$

Section - C

2 x 16 = 32

Note : Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) Solve by using Laplace Transform

$\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}$, subject to conditions

$u(x,0) = 0, u(3,t) = 0, u_x(0,t) = 0$ and

$u_t(x,0) = 12\cos\pi x + 16\cos 3\pi x - 8\cos 5\pi x$

11) Use the method of Fourier Transform to determine the displacement $y(x,t)$ of an infinite string, given string is initially at the rest and that initial displacement is $f(x), -\infty < x < \infty$, Show that the solution can also be put in the form $y(x, t) = \frac{1}{2}[f(x+ct) + f(x-ct)]$

12) Using Hilbert-Schmidt method prove that given integral equation has unique solution and find its solution.

$$g(x) = (x+1)^2 + \int_{-1}^{1} k(x, t)\, g(t)\, dt$$

Where $k(x, t) = (xt + x^2 t^2)$

13) (i) Use Laplace transform to solve:

$(D^2 + n^2)y = a\sin(nt + \alpha)$ given at $t = 0, y = 0$

and $\frac{dy}{dt} = 0$

(ii) Solve the integral equation.

$$g(x) = 1 + \int_{0}^{1} (1 + e^{x+t}) g(t)\, dt$$
